

Optimal design of a solar driven heat engine based on thermal and thermo-economic criteria



Mohammad H. Ahmadi^{a,*}, Saeed Dehghani^a, Amir H. Mohammadi^{b,c,*}, Michel Feidt^d, Marco A. Barranco-Jimenez^e

^a Faculty of Mechanical Engineering, K.N. Toosi University, Tehran, Iran

^b Institut de Recherche en Génie Chimique et Pétrolier (IRGCP), Paris Cedex, France

^c Thermodynamics Research Unit, School of Engineering, University of KwaZulu Natal, Howard College Campus, King George V Avenue, Durban 4041, South Africa

^d Laboratoire d'Energétique et de Mécanique Théorique et Appliquée, ENSEM, 2, avenue de la Forêt de Haye, 60604 54518 Vandoeuvre, France

^e Departamento de Ciencias Básicas, Escuela Superior de Computo del IPN, Av. Miguel Bernard Esq. Juan de Dios Batiz U.P., Zacatenco CP 07738, D.F., Mexico

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ABSTRACT

To study the optimum design factors and the optimal thermo-economical performances an optimal performance analysis of a solar driven heat engine system at maximum thermo-economical objective function conditions is done numerically. In the present investigation, thermodynamic analysis and NSGAII algorithm are employed to optimize objective function associated to the power output, thermal efficiency for a Solar driven engine system. Three decision-making procedures are applied to optimized answers from the results. The error through investigation is shown using error analysis.

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1. Introduction

Performance optimization studies of heat engines using finite-time thermodynamics were begun by Chambadal [1] and Novikov [2] and were continued by Curzon and Ahlborn [3]. At first step, the performance of an endoreversible Carnot heat engine at maximum power output are scrutinized by Curzon and Ahlborn [3]. Over the recent decade, great number of optimization investigations for heat engines based on endoreversible and irreversible models have been done owing to finite-time and finite-size constraints under various heat transfer conditions, commonly linear and non-linear ones. The eager authors could refer to a literature survey compiled by Bejan [4], Chen et al. [5] and Durmayaz et al. [6]. The power output, thermal efficiency, entropy generation, the ecological benefit and thermo-economic objectives are often selected as optimization factors. Great attention has been drawn to optimization of the solar-driven heat engines based on various performance criteria [7–21] where efficiency, power output and power density are often selected as the objective functions for optimization. The optimum

operation conditions of an endoreversible and irreversible solar driven heat engine was studied by Sahin [17,18] and Sogut and Durmayaz, respectively [20].

Sahin et al. [22], employed this method to study the thermoeconomics of an endoreversible heat engine in terms of the maximization of a profit function defined as the quotient of the power output and the annual investment cost. Recently, Barranco-Jiménez et al. [23], analyzed the optimum operation conditions of an endoreversible heat engine with different heat transfer laws at the thermal couplings but operating under maximum ecological function conditions. Furthermore, the thermoeconomic optimum operation conditions of a solar-driven heat engine have been investigated by Barranco-Jiménez et al. [24].

In these studies, they considered three regimes of performance: (i) The maximum power regime (MPR) [25–27], (ii) the maximum efficient power [28,29], and (iii) the maximum ecological function regime (MER) [30,31].

In this work, we study the thermoeconomics of an irreversible heat engine by considering losses due to heat transfer across finite time temperature differences [32–35], heat leakage between thermal reservoirs [36–44] and internal irreversibilities [45–47] in terms of a parameter which comes from the Clausius inequality. In this work, two regimes of performance are considered: The maximum power regime, and the ecological function regime. In

* Corresponding authors. Address: Institut de Recherche en Génie Chimique et Pétrolier (IRGCP), Paris Cedex, France (A.H. Mohammadi).

E-mail addresses: a.h.m@irgcp.fr (M.H. Ahmadi), mohammadhosein.ahmadi@gmail.com (A.H. Mohammadi).

Nomenclature

A	area
a, b	investment cost proportionality coefficient
C_i	annual investment cost
F_{ij}	objectives for solutions of the Pareto frontier
F_{ij}^n	non-dimensionalized objective
F_M	objective function associated to the power output
f	relative investment cost
W	output power
$\dot{Q}$	heat transfer rate
T	temperature
T_a	the temperature of the fluid surrounding the body
U	heat transfer coefficient

Greek letter

α	thermal conductance
β	ratio of the heat conductance parameter
τ	heat sources temperature ratio
η	efficiency
γ	internal conductance
ξ	ratio of internal conductance

$$\theta = \frac{T_h}{T_H}$$

Subscripts

H	related to high temperature thermal reservoir
L	related to low temperature thermal reservoir
h	hot side
c	cold side
R	non-endoreversibility parameter
LK	leakage
HT	total heat rate

Acronyms

NSGA	Nondominated Sorting Genetic Algorithm
MPR	maximum power regime
MER	maximum ecological regime
EA	evolutionary algorithm
MOEAs	multi-objective evolutionary algorithms

addition, in the present work, thermo-economical analysis, conductive–convective and radiative terms are considered through a heat transfer law of the Dulong–Petit type [48,49].

Multi-objective optimization has supreme presence in different engineering problems such as skyline computation, vehicle routing issues and so forth [53–55]. Solution of the multi-objective optimization issues is an acutely hard purpose which needs the contemporaneous satisfaction of a number of different and sometimes conflicting objectives. Evolutionary algorithms (EA) were expanded and applied during the 18th century in an effort to stochastically solve problems of this generic class [56]. A reasonable solution to a multi-objective quandary is to inquiry a set of solutions, each of them satisfies the objectives at plausible level without being overcome by any other solution [57]. Multi-objective optimization quandaries universally represent a possibly uncountable collection of solutions so-called Pareto frontier, whose evaluated vectors illustrate the best feasible trade-offs in the objective function space. In this case, multi-objective optimization of different thermodynamic and energy systems have been drawn attention by investigators these days [58–65]. In the present investigation, multi-objective optimization is conducted to achieve the optimal performance analysis of a solar driven heat engine system at maximum ecological objective function conditions. In our study, we consider different heat transfer laws in the thermal couplings and several decision variables such as temperature ratio, heat source temperature, hot working fluid temperature, ratio of heat transfer areas (heat sink to heat source) are considered. The presented analysis is great at designing and suitable design parameters can be obtained based on the analysis. The paper is organized as follows: In Section 2, we present the thermodynamic description of the system, we apply the first and second laws of thermodynamics to construct the objective functions, in Section 3 we describe the multi-objective optimization using evolutionary algorithms, in Section 4, we describe our decision-making methods used in our work, in Section 5 we present our main results and finally in Section 6, we present our conclusions.

2. Theoretical model

The considered irreversible solar-driven heat engine operates between a heat source of temperature T_H (the collector) and a

heat sink of temperature T_L (cooling water), see Fig. 1a. Working fluid temperature which exchange heat with reservoir at temperature T_H is considered as T_h . Similarly, working fluid temperature which exchanges heat with reservoir at temperature T_L is defined as T_c . A T – S diagram of the model including heat leakage, finite time heat transfer and internal irreversibilities is also shown in Fig. 1b.

An attempt to describe combined conductive–convective and radiative cooling by a power-law relationship is given by the so-called Dulong–Petit law of cooling [23], which is

$$\dot{Q} = UA(T_a - T)^n \quad (1)$$

where $\dot{Q}$ is the rate of heat loss per unit area from a body at temperature T , T_a is the temperature of the fluid surrounding the body, and n is an exponent with a value between 1.1 and 1.6 [23]. Some authors have established that $n = 5/4$ based on studies made by Lorentz and Langmuir. In the present paper, we use the DP law of cooling with $n = 5/4$.

The rate of heat leakage $\dot{Q}_{LK}$ from the hot reservoir at temperature T_H to the cold reservoir at temperature T_L with thermal conductance γ is given by [48]

$$\dot{Q}_{LK} = \gamma(T_H - T_L)^{5/4} = \xi U_H A_H (T_H - T_L)^{5/4} \quad (2)$$

where γ is the internal conductance of the heat engine and ξ denotes the ratio of the internal conductance with respect to the hot-side convection heat transfer coefficient and heat transfer area, that is, $\xi = \frac{\gamma}{U_H A_H}$ [47]. The $5/4$ exponent is usual in a Dulong–Petit heat transfer law [48]. The rate of heat flow from the hot source to the heat engine is given by,

$$\dot{Q}_H = U_H A_H (T_H - T_h)^{5/4} \quad (3)$$

where U_H is transfer coefficient and A_H is the heat transfer area of the hot side heat exchanger. Eqs. (2) and (3) are of the Dulong–Petit type, including conductive–convective and radiative effects [48] rather than only use a Stefan–Boltzmann law. Note that, Stefan–Boltzmann law is proper for space applications [31], where the effect of atmospheric gases is not present. On the other hand, a Newtonian heat transfer is presumed as the main mode of heat transfer to the low temperature reservoir, so the heat flow rate from the heat engine to the cold reservoir, $\dot{Q}_L$, can be calculated as,

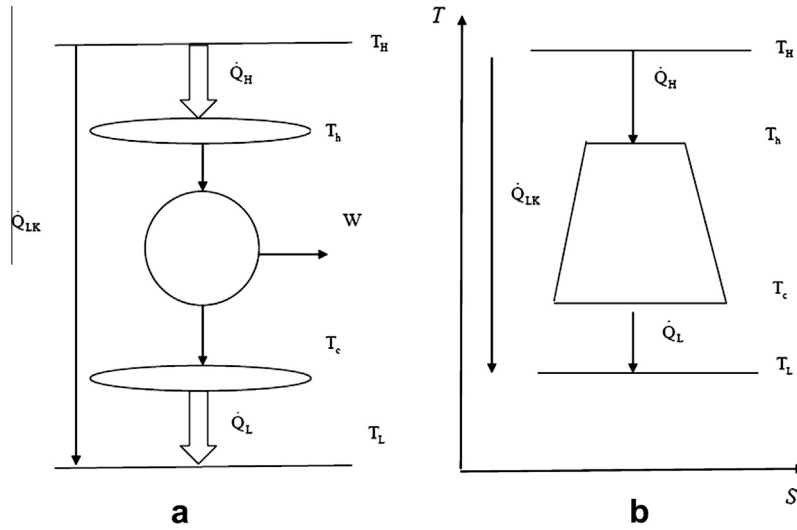


Fig. 1. Schematic diagram of the irreversible heat engine and its T - S diagram.

$$\dot{Q}_L = U_L A_L (T_c - T_L)^{\frac{5}{4}} \quad (4)$$

where U_L is the cold side heat transfer coefficient and A_L is the heat transfer area of the cold side heat exchanger. Then, the total heat rate $\dot{Q}_{HT}$ transferred from the hot reservoir is,

$$\dot{Q}_{HT} = \dot{Q}_H + \dot{Q}_{LK} = U_H A_H (T_H - T_h)^{\frac{5}{4}} + \xi U_H A_H (T_H - T_L)^{\frac{5}{4}} \quad (5)$$

and the total heat rate $\dot{Q}_{LT}$ transferred to the cold reservoir is,

$$\dot{Q}_{LT} = \dot{Q}_L + \dot{Q}_{LK} = U_L A_L (T_c - T_L) + \xi U_H A_H (T_H - T_L)^{\frac{5}{4}} \quad (6)$$

Applying the first law of thermodynamics, the power output is given by,

$$W = \dot{Q}_{HT} - \dot{Q}_{LT} = U_H A_H (T_H - T_h)^{\frac{5}{4}} - U_L A_L (T_c - T_L) \quad (7)$$

Applying the second law of thermodynamic to the irreversible part of the model we get,

$$\oint \frac{dQ}{T} = \frac{\dot{Q}_H}{T_h} - \frac{\dot{Q}_L}{T_c} < 0 \quad (8)$$

One can rewrite the inequality in Eq. (7) as,

$$\frac{\dot{Q}_H}{T_h} = R \frac{\dot{Q}_L}{T_c} \quad (9)$$

where R is the so-called non-endoreversibility parameter [50,51]. Substituting Eqs. (3) and (4) into Eq. (9), a relationship between T_c and T_h is obtained as,

$$\frac{T_c}{T_L} = \frac{R A_R}{\left(R A_R - \beta \frac{(1-\theta)^{\frac{5}{4}}}{\theta} \right)} \quad (10)$$

where $\theta = \frac{T_h}{T_H}$, A_R and β are the ratios of the heat transfer areas and the heat conductance parameter respectively, and are defined as,

$$A_R = \frac{A_L}{A_H} \quad (11)$$

$$\beta = \frac{U_H}{U_L} T_h^{\frac{1}{4}} \quad (12)$$

These two parameters can be taken as design parameters. The thermal efficiency of the irreversible heat engine is,

$$\eta = 1 - \frac{\dot{Q}_{LT}}{\dot{Q}_{HT}} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H + \dot{Q}_{LK}} \quad (13)$$

In thermoeconomic analysis of power plant models, an objective function can be specified in terms of a characteristic function (power output [18,26,52], ecological function [24,30,49], etc.) and the cost involved in the performance of the system.

In this context, De Vos [26] studied the thermoeconomics of a Novikov power plant model in terms of the maximization of an objective function expressed as the quotient of the power output and the operating costs of the plant.

In that paper, De Vos considered a function of costs with two parts: (i) The cost of the investment which is assumed as proportional to the size of the plant and (ii) the cost of the fuel consumption which is assumed to be proportional to the heat input in the Novikov model. Similarly, Sahin and Kodal base on Curzon and Ahlborn [25] model, formulated a thermoeconomic analysis in terms of an objective function. Their function defined as power output per unit total cost including both the investment and fuel costs [18], but assuming that the size of the plant can be taken as proportional to the total heat transfer area, instead of the maximum heat input which previously considered by De Vos [26].

Following the Sahin et al. procedure [52], the objective function has been defined as the power output per unit investment cost due to a solar driven heat engine does not consume fossil fuels. To optimize power output per unit total cost, the objective function is defined by [52] as follows:

$$F = \frac{W}{C_i} \quad (14)$$

where C_i represents annual investment cost of plant. C_i is assumed to be proportional to the size of the plant. Also, the size of the plant can be proportional to the total heat transfer area. Consequently, the annual investment cost of the system can be written as [52],

$$C_i = a A_H + b A_L \quad (15)$$

where the investment cost proportionality coefficients for the hot and cold sides a and b respectively are equal to the capital recovery factor times investment cost per unit heat transfer area, and their dimensions are $ncu/(year \ m^2)$, ncu being the national current unity. By using Eqs. (3), (7), (10), (14), and (15), we get a normalized expression for the objective function associated to the power output given by [52],

$$F_M = \frac{\overline{W}}{\overline{C}} = \frac{\tau(1-\theta)^{\frac{1}{4}} - \frac{A_R}{\beta} \left(\frac{T_c}{T_L} - 1 \right)}{\frac{A_R}{\beta} \left(\frac{1-f}{f} \right) + 1} \quad (16)$$

where $\tau = \frac{T_H}{T_L}$ and the parameter f , is the relative investment cost of the hot size heat exchanger defined as [52],

$$f = \frac{a}{a+b} \quad (17)$$

And, by using Eqs. (2)–(4), the thermal efficiency, η , of the irreversible heat engine can be expressed by,

$$\eta = \frac{\tau\beta(1-\theta)^{\frac{5}{4}} - A_R\left(\frac{T_c}{T_L} - 1\right)}{\tau\beta(1-\theta)^{\frac{5}{4}} + \zeta\tau\left(\frac{\tau-1}{\tau}\right)^{\frac{5}{4}}} \quad (18)$$

3. Multi-objective optimization with evolutionary algorithms

3.1. General concepts of the multi-objective optimization

Imagine a decision-maker that would like to optimize objectives which are non-commensurable. The decision-maker does not know the priority of the objectives. In addition, all objectives are of the minimization kind. For a minimization type of objectives, it can be altered to the maximization type by multiplying negative one. A minimization (optimization) multi-objective decision problem with K objectives is defined as follows.

Given a non-dimensional decision variable vector $x = \{x_1, \dots, x_n\}$ in the solution area X , find a vector x^* which minimizes a collection of K objective functions $f(x) = \{f_1(x^*), \dots, f_k(x^*)\}$. The solution space X is restricted by a number of constraints, such as $g_j(x^*) = b_j$ for $j = 1, \dots, m$, and bounds on the decision variables [57]. Totally, it is not possible to minimize all the K objective functions simultaneously with one solution vector X which means in real-world problems, objectives under consideration conflict each other. Thus, optimizing X with concerning a uni-objective mostly leads to unreasonable consequence with respect to the other objectives. Therefore a novel implication, so-called the “Pareto optimum solution”, is applied in multi-objective optimization quandaries. A practical solution X is called “Pareto optimal” if there is no other plausible solution Y that prevails solution X . Therefore, a feasible solution Y is entitled to overcome another conceivable solution X , if and only if, $f_i(Y) \leq f_i(X)$ for $i = 1, \dots, k$ and $f_j(Y) < f_j(X)$ for at least one objective function which explains that a possible vector X is denominated Pareto optimal if there is no other possible solution Y that would decrease some objective functions without concluding a simultaneous increase in minimum one other objective function. The collection of all plausible non-dominated solutions in X is led to the “Pareto optimal set”, and for a certain Pareto optimal

collection, the corresponding objective function values in the objective space are denominated the “Pareto optimal frontier” [57].

Fig. 2 represents the objective functions space for an optimization problem with two objective functions f_1 and f_2 . The feasible area is insight and infeasible area is out of the curve. Each point in the feasible area denotes a solution. It is obvious that the values of functions f_1 and f_2 for point M are lower than the corresponding values of point J . Therefore point M overcomes point J or point M is better than point J . In the same way points L and N dominate M . But points I and K neither overcome M nor are overcome by M . Thus, only the points that are placed in the left-down parts of M would overcome M . There is not any point in the feasible space placed in the left-down part of point R . Thus R is not overcome by the other points in the feasible space and hence R is a Pareto optimal. This is true for points P, A, R, E, T, O and all of the other points located at the bold curve interpreted as “The Pareto Optimal Frontier” in Fig. 2. In the feasible space, the minimum value of f_1 is for P and the minimum value of f_2 is for point O . Thus P and O are the solutions for one objective optimization issues whose objective functions are f_1 and f_2 , respectively. The other points on the Pareto frontier are also optimal, but a decision-making procedure is needed to find which of them should be chosen.

3.2. Optimization via EA

In the present investigation, the Pareto frontier is obtained using the Genetic Algorithm (GA) which is identified as a field of evolutionary algorithm. Genetic algorithms were evolved by John Holland in the 1960s to import the mechanisms of natural adaptation into computer algorithms and numerical optimization [59]. They are employed as a computer simulation in a way that a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or pheno-types) to an optimization problem evolves toward better solutions. It commences from a population of randomly produced exclusives and occurs in generations. In each generation, the fitness of every individual is measured; multiple individuals are stochastically opted from the current population, and altered to form a new population. The new group is employed in the next iteration. Generally, the algorithm finishes when either a maximum number of generations have been produced, or a satisfactory fitness level has been accomplished for the population. If the algorithm has ended owing to a maximum number of generations, a satisfactory solution might have been reached or might not have been gained. In genetic algorithms, a candidate solution to a problem is named a chromosome, and the evolutionary viability of each chromosome is denoted by a fitness function. This method can be applied to optimize non-linear problems [57,60].

Multi-objective evolutionary algorithms (MOEAs) have been evolved in the recent decades by a lot of tests on complex mathematical quandaries. On real-world it has shown that they can omit

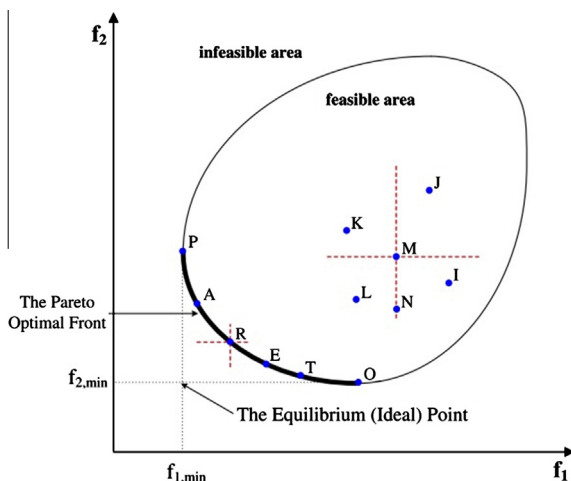


Fig. 2. Schematic of the objectives space.

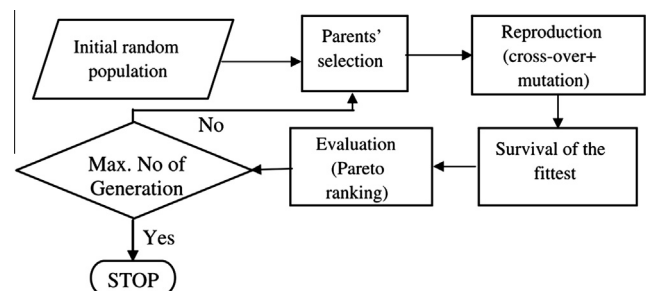


Fig. 3. Scheme for the multi-objective evolutionary algorithm used in the present study.

the obstacles of classical methods [57,60]. The structure of the MOEA exploited in this investigation is depicted in Fig. 3 [59]. The real values of decision variables are applied instead of their binary coded.

3.3. Objective functions, decision variables and constraints

Two objective functions for this study are the thermal efficiency (η) and the objective function associated to the power output (F_M) respectively.

In this paper five decision variables have been considered as follows:

- θ : temprature ratio ($\theta = \frac{T_h}{T_c}$),
- A_R : ratio of the heat transfer areas,
- τ : temprature ratio ($\frac{T_h}{T_c}$),
- f : the relative investment cost of the hot size heat exchanger,
- β : ratio of the heat conductance parameter.

The objective functions with respect to following constraints have been solved:

$$0.4 \leq \theta \leq 0.9 \quad (19)$$

$$2 \leq A_R \leq 4 \quad (20)$$

$$3 \leq \tau \leq 5 \quad (21)$$

$$0.5 \leq f \leq 0.9 \quad (22)$$

$$0.5 \leq \beta \leq 1.5 \quad (23)$$

4. Decision-making in the multi-objective optimization

In multi-objective optimization a process of decision-making for selection of the final optimal solution from available solutions is needed. There are a number of methods for decision-making process in decision problems. These methods can be utilized for selection of a final optimum solution from the Pareto frontier. Since, dimension of various objectives in a multi-objective optimization problem might be different, therefore, before any decision, dimensions and scales of objective space should be unified. In this regard, objectives vectors should be non-dimensionalized before the decision-making process. There are various methods of non-dimensionalization employed in decision making including Euclidean non-dimensionalization, and fuzzy non-dimensionalization.

4.1. Non-dimensionalization methods

4.1.1. Euclidean non-dimensionalization

The matrix of objectives for various solutions of the Pareto frontier is denoted by F_{ij} where i is an index for each solution on the Pareto frontier and j is an index for each objective in objective space. In this method, a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\sqrt{\sum_{i=1}^m (F_{ij})^2}} \quad \text{For minimizing and maximizing objectives} \quad (24)$$

4.1.2. Fuzzy non-dimensionalization

In this method, a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij} - \min(F_{ij})}{\max(F_{ij}) - \min(F_{ij})} \quad \text{For maximizing objectives} \quad (25a)$$

$$F_{ij}^n = \frac{\max(F_{ij}) - F_{ij}}{\max(F_{ij}) - \min(F_{ij})} \quad \text{For minimizing objectives} \quad (25b)$$

In this paper, most familiar and usual type of decision-making processes including the fuzzy Bellman–Zadeh, LINMAP and TOPSIS method are applied in parallel and the final optimal solution is selected using these three methods. The Bellman–Zadeh method utilizes the fuzzy non-dimensionalization while the LINMAP and TOPSIS methods utilize Euclidean non-dimensionalization. The following sections are described these decision-making algorithms.

4.2. Decision making methods

4.2.1. Bellman–Zadeh decision-making method

A final decision is defined by the Bellman and Zadeh model as the intersection of all fuzzy criteria and constraints and is represented by its membership function. In this method, a matrix of membership function is defined. Column of this matrix includes the fuzzy membership function for each objective. Rows of the membership matrix reflect magnitudes of the membership functions for each solution of the Pareto frontier. Therefore, the number of columns in the membership function matrix is equal to the number of objectives and the number of rows is equal to the number of solutions located on the Pareto frontier. Detail for method of definition for the membership function has been presented in Ref. [66]. The fuzzy membership function for each solution is set at the minimum value of the membership functions of all objectives for the proposed solution. Therefore, a vector of membership is obtained in which its elements are the minimum membership function of objectives at each solution. Finally, the maximum of these minimum, i.e. an element of the membership vector with

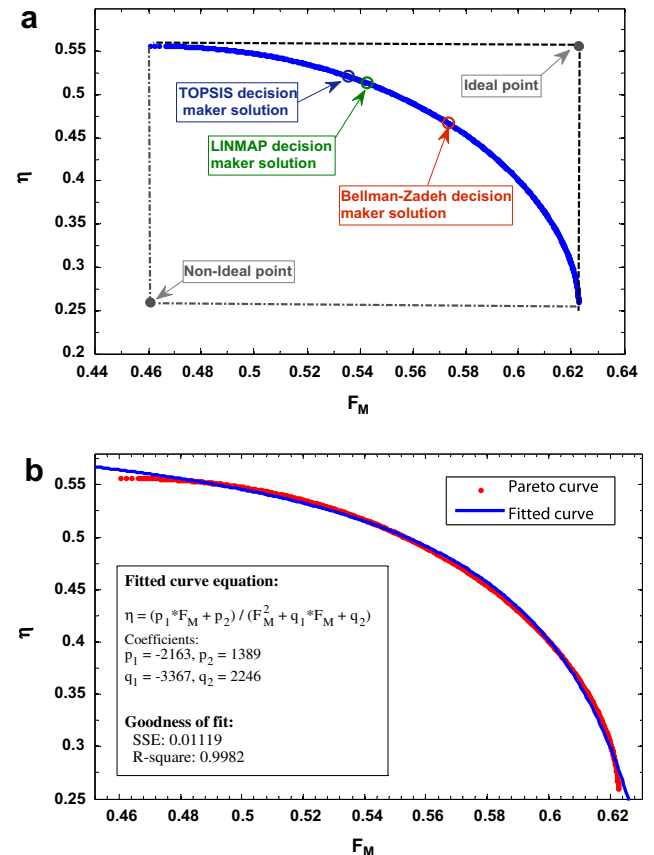
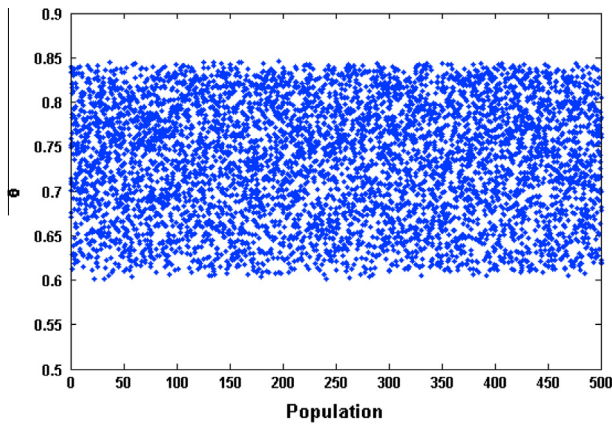
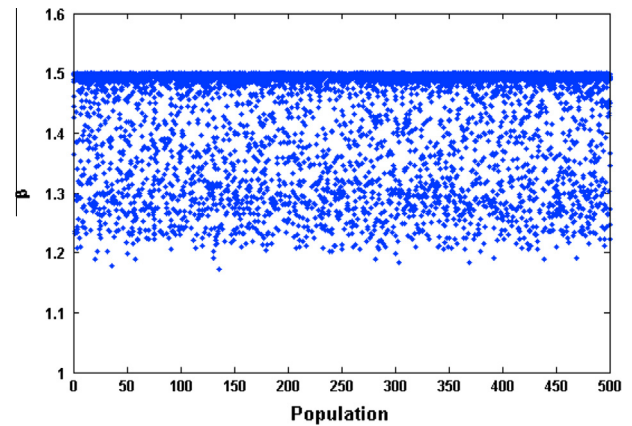
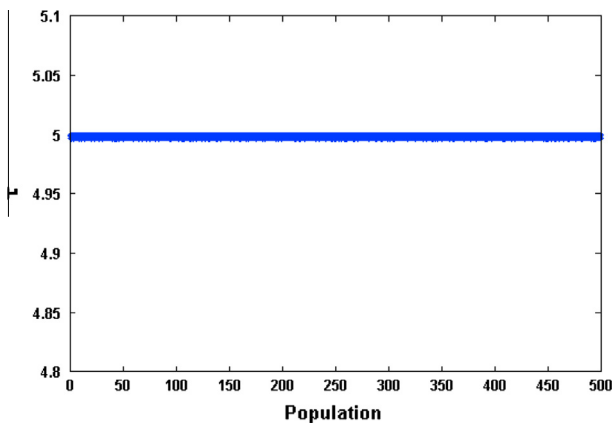
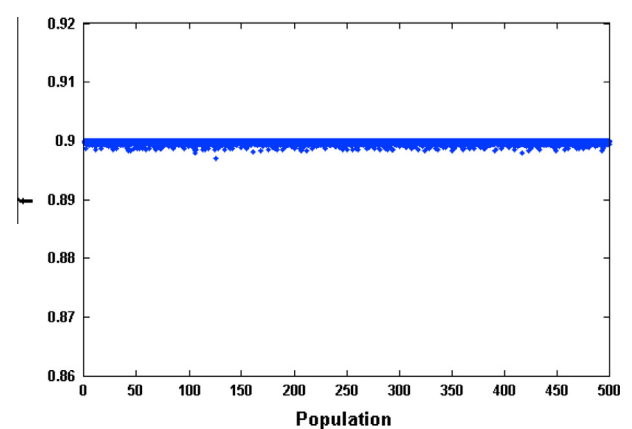
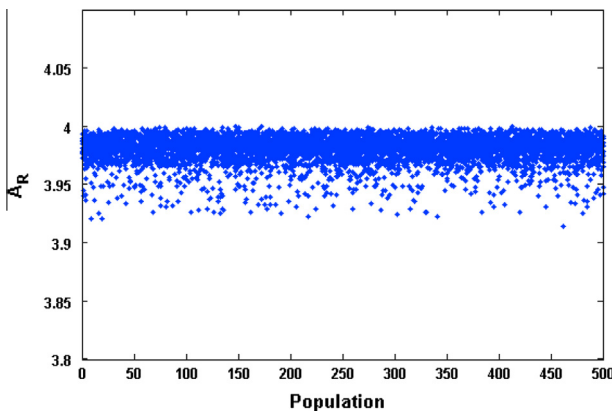


Fig. 4. (a and b) Pareto optimal frontier in objectives' space.

Fig. 5. Scatter distribution of θ with population in Pareto frontier.Fig. 8. Scatter distribution of β with population in Pareto frontier.Fig. 6. Scatter distribution of τ with population in Pareto frontier.Fig. 9. Scatter distribution of f with population in Pareto frontier.Fig. 7. Scatter distribution of A_R with population in Pareto frontier.

maximum values of membership function is selected as a final solution. In other word, in the Bellman–Zadeh fuzzy decision making a maximum of minimums (minimum membership function of objectives) is selected as the final optimal solution. In Ref. [66] more detail is presented.

4.2.2. LINMAP decision-making method

An ideal point on the Pareto frontier is a point in which each objective is optimized regardless to the satisfaction of other objectives. It is clear that in the multi-objective optimization it is

impossible to have each objective in its optimal condition that it can obtain in a single-objective optimization. Consequently, the ideal point is not located on the Pareto frontier. In the LINMAP method, after Euclidean non-dimensionalization of all objectives, the spatial deviation of each solution on the Pareto frontier from the ideal point is determined. Finally, a solution with a minimum spatial distance from the ideal point is selected as a desired final optimal solution. More details of the LINMAP method have been presented in Ref. [66].

4.2.3. TOPSIS decision-making method

In this method, beside the ideal point, a “non-ideal point” is defined. The non-ideal point is the ordinate in objective space in which each objective has its worst value. Therefore, beside the solution spatial distance from the ideal point, the solution spatial distance from the non-ideal is utilized as a criterion for selection of the final solution. In the TOPSIS method a solution with a maximum distance from the non-ideal point and minimum distance from the ideal point is selected as a desired final optimal solution. More details of the TOPSIS method have been given in Ref. [66].

5. Results and discussion

The objective function associated to the power output and thermal efficiency of the solar driven engine are maximized simultaneously using the multi-objective optimization method which works based on the NSGA-II algorithm.

Table 1
Decision making of multi-objective optimal solutions.

Decision maker	Decision variables					Objective functions	
	θ	A_R	τ	f	β	η	F_M
TOPSIS	0.671637	3.997406	4.99973	0.89996	1.326286	0.5356731	0.52153751
LINMAP	0.679862	3.990805	4.999976	0.899967	1.336159	0.542754	0.513691706
Bellman–Zadeh	0.722324	3.992746	4.99979	0.899942	1.472895	0.5735338	0.46678106

In this regard, optimization is performed with objective functions that are expressed by Eqs. (16) and (18) constraints which are expressed with Eqs. (19)–(23).

Design parameters (decision variables) of optimization are the temperature ratio ($\theta = \frac{T_h}{T_c}$), ratio of the heat transfer areas, temperature ratio ($\tau = \frac{T_h}{T_c}$), the relative investment cost of the hot size heat exchanger, ratio of the heat conductance parameter.

Pareto optimal frontier for two objective functions, objective function associated to the power output, thermal efficiency of Solar driven engines are represented in Fig. 4a. The chosen points based on decision making methods are shown, too. The fitted curve equation of the Pareto optimal frontier is obtained and illustrated in Fig. 4b, which can be applied to gain system thermal efficiency, as following:

$$\eta = \frac{(-2163F_M + 1389)}{(F_M^2 - 3367F_M + 2246)} \quad (25c)$$

$$R^2 = 0.9982$$

$$0.46 \leq F_M \leq 0.62$$

$$0.25 \leq \eta \leq 0.55$$

Figs. 5–9 demonstrate the scattered distribution of the design variables. These plots can provide better vision of variation of the design variables from the Pareto frontier. According to this distribution, it is found that heat sources temperature ratio and relative investment cost reach their maximum values. It displays that increase in these two design parameters result in improvement in both objective functions. Consequently, their maximum values are chosen.

The optimal results for decision parameters and objective functions using LINMAP, TOPSIS, Bellman–Zadeh decision-making methods are summarized in Table 1.

Maximum and mean error analyses for objective functions are categorized in Table 2. The analysis is done for all mentioned decision-making methods.

First row of Table 2 express maximum absolute percentage error of three decision making methods and second row of this table report mean absolute percentage error of each decision making methods. In both cases errors are less than 1%.

The mean absolute percentage error was calculated in the error analysis of implementing methods. To assess this goal, 30 runs of computer code were performed and a final solution selected by fuzzy Bellman–Zadeh, LINMAP and TOPSIS decision making methods were obtained for each run. Absolute percentage error of solutions obtained in each run was calculated comparing to the best results introduced as the final solution in this paper.

Table 2
Error analysis based on the mean absolute percent error (MAPE) method.

Errors of decision makers	LINMAP		TOPSIS		Bellman–Zadeh	
	η	F_M	η	F_M	η	F_M
Max error %	0.05	0.29	0.39	0.23	0.19	0.10
Average error %	0.03	0.20	0.15	0.11	0.11	0.06

6. Conclusions

Throughout current research work enormous efforts were made to figure out the optimum condition of the two addressed objective functions such as thermal efficiency and the objective function associated to the power output for a solar driven heat engine. In developed multi-objective optimization approach five different variables including temperature ratio ($\theta = \frac{T_h}{T_c}$), temperature ratio ($\tau = \frac{T_h}{T_c}$), ratio of the heat transfer areas (A_R), ratio of the heat conductance parameter (β), the relative investment cost of the hot size heat exchanger (f) were considered as decision variables. Due to obtained solutions from different decision making approaches, TOPSIS decision making has higher maximum error percent than other two addressed decision making approaches such as LINMAP and Fuzzy approaches. The results of the presented thermodynamic model for solar driven engine implementing NSGAII algorithm are acceptable.

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